POSSIBLE STEADY-STATE VOLTAGE STABILITY ANALYSES OF ELECTRIC POWER SYSTEMS

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ABSTRACT

This paper deals with steady-state voltage stability problem as one of the main topics of interest in today's power system operation and control worldwide. It mainly focuses on analytical description of the problem and conventional (analytical) solution of simple power systems using the theory of load flow analysis. Secondary, it introduces so-called Continuation load flow analysis, which is currently used for evaluation voltage stability margins, volume of available reactive power reserves and location of weak buses/areas of the power system sensitive to load increase.

KEYWORDS

Voltage stability, V-P curve, saddle-node bifurcation point, Continuous load flow analysis (CLF), Predictor/Corrector method.

1. INTRODUCTION

Nowadays, the majority of electric power systems are operated relatively close to their operational limits. Due to several factors, especially the strong representation of renewable power sources and deregulated electricity market policies, the voltage stability has been considered as the top-priority topic for avoiding related voltage collapse and black-out/islanding problems.

Steady-state stability is defined as the capability of the network to withstand a small disturbance (fault, small change of parameters, topology modification) in the system without leaving a stable equilibrium point. On the contrary, voltage instability can be described as the system state, when the voltage slowly decreases (due to insufficient reactive reserves) until significant voltage drop appears (voltage collapse). Such a situation may occur in a relatively large time frame - from tens of seconds to several minutes. Voltage collapse may arise as a result to a combination of different negative factors, such as load increase, long electrical distances between reactive power sources and the loads, too low source voltages, crucial changes in network topology and no reactive compensations available.

Inappropriate voltage profiles are usually averted by the action of ULTC transformers in the network. However, each tap position corresponds to an increase of the load eventually leading to higher branch losses and further voltage drop. Therefore, the ULTC transformers are blocked during low voltage situations.

In case of paralelly-connected lines, tripped line (during the fault) would result in double series impedance but only half shunt susceptance. In consequence, reduced amount of reactive power is delivered for supporting locally bus voltage magnitudes due to higher var losses in the branch and lower var injections through the shunt lines.

When a generator reaches its var limit, local voltage is forced to decrease, BV^2 system injections are curtailed and branch current rises due to constant power generation. As a result, branch var losses increase causing even worse local voltage profiles and an infinite loop continues until the voltage collapse occurs. Therefore, control centres must send an incentive to affected power sources to increase the var generation, switch on available compensation devices and perform load shedding of selected customers.

Especially dangerous is the situation in induction motors (so-called stalling). When restoring low supply voltage (after the fault), motors start to accelerate consuming large amount of current, which eventually leads to line tripping. The effect is even lower voltage, repetitive occurrence of stalling and line trippings.

For the prevention of voltage collapse, several types of compensation devices are massively used - both shunt capacitors and inductors, series capacitors (TCSCs), SVCs, synchronous condensers, STATCOMs, etc. To reduce the voltage profile (in case of low demand), var consumption must be increased - switching in shunt reactors, disconnecting cable lines, reducing MVAr output from generators and synchronous condensers, tap changing, etc. Opposite actions are taken for increasing bus voltages.

However, voltage conditions may be even worsened in some cases by massive use of shunt capacitors.

2. ANALYTICAL SOLUTION USING THE THEORY OF LOAD FLOW ANALYSIS

2.1. Theoretical background to the nose (voltage-power, V-P) curve

Major goal of the steady-state voltage stability analysis is the computation of the V-P curve, also known as the "nose curve" – see Fig. 1. For the chosen PQ bus of the network, this curve shows the dependence of the bus voltage magnitude (vertical axis) on the active power load in the bus when increased from zero to its maximum possible value. Increase of active power load with a zero reactive power load or of both active and reactive power loads with constant power factor can be realized.

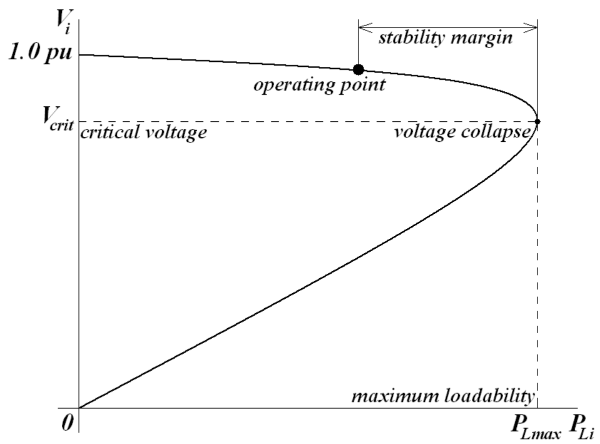

Figure 1 – The V-P curve for bus *i* of a general power system

At the beginning, the voltage starts from the value equal or close to 1.0 per units slowly moving down with gradually increased active power load. In a certain point – so called "saddle node bifurcation point" or "singular point", further load increase would provide no feasible operating voltage magnitude at all and voltage collapse occurs. Therefore, this limited load value is called the "maximum loadability" and corresponding voltage value the "critical voltage". From this point on, only the decrease of power leads to a solution until zero voltage and load are reached. In the practice, only the higher half of bus voltage magnitudes provides a stable operating point. The other one is in the unstable region, which produces the low-voltage solutions under undesirably high current conditions. Note: Term "bifurcation" simply relates to the fact, that from the singular point there are two different voltage magnitudes for each active power load value.

One of the most important outputs is the determination of the voltage stability margin. It simply provides the information about how far the current operating point of the network is from the voltage collapse. Among others, level of the reactive power reserve shows the amount of var compensation available in individual steps of the V-P curve.

The impact of various compensation devices on the V-P curve is demonstrated in Fig. 2. As can be seen, the use of shunt capacitor eventually leads to higher maximum loadability value, while it can be decreased using the shunt inductor. Therefore, the customers may be charged for significantly high reactive power demands (leading pf.) when reducing the voltage stability margin of the power system.

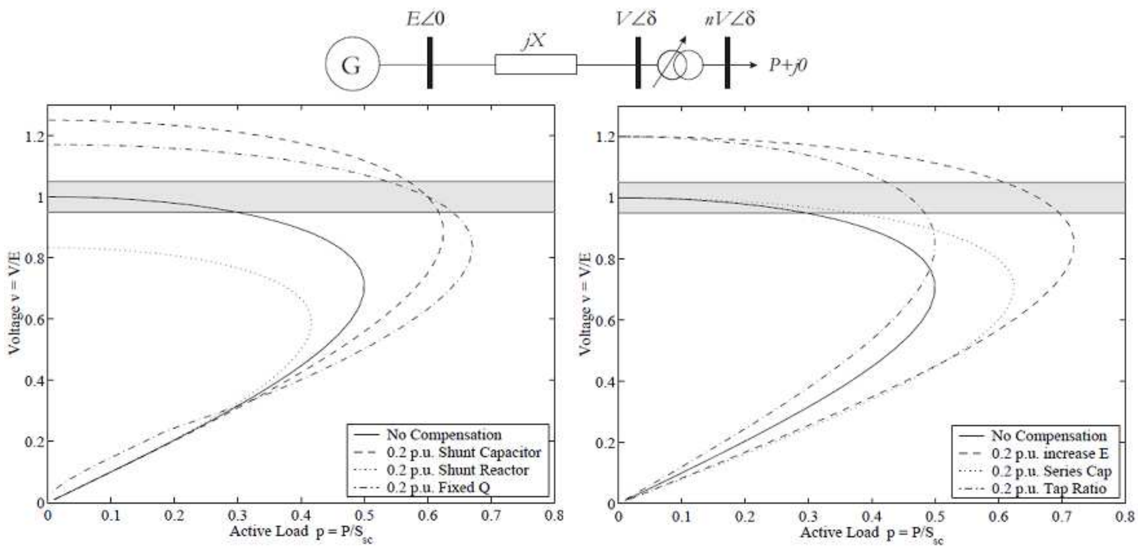

Figure 2 – Effects of possible compensation devices on the V-P curve [3]

As shown in Fig. 2 above, higher critical voltage value along with higher maximum loadability can be suitably reached using series capacitors, fixed var injections, increased supply voltage and tap ratio switching actions.

2.2. Analyzed problem I. (3-bus power system)

Although the voltage collapse is a dynamic problem, it can be analyzed using static methods whether the power system parameters move slowly. Therefore, even the symbolic-complex method can be used for analytical description and solution of the problem.

As the first case study, a 2-bus power system (see Fig. 3) has been considered. The network consists of 3 buses in total, where bus 3 is the consumption point with connected P-Q load. Bus 3 is connected to bus 2 via a long transmission line (R = 8.2 Ω, X = 89.63 Ω, B = 1258 μS). The load is supplied from a large part of the country's transmission network, which has been simplied by the Thevenin's theorem into an equivalent power source in slack bus 1. Bus 1 is interconnected with the transmission line 2-3 via relevant system reactance (X = 36 Ω). The task is to derive the analytical relations between the active power load, voltage magnitude and the phase angle in bus 3.

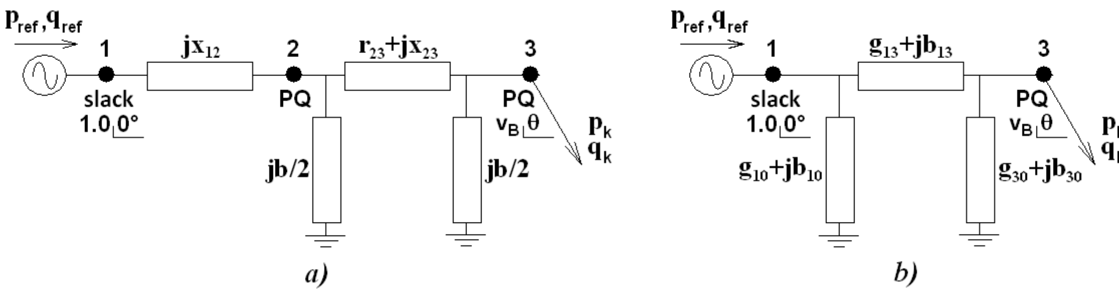

Figure 3 – Original network model a) and its final representation b)

For the voltage stability study, calculation in per units (V_{base} = 400 kV, S_{base} = 100 MVA) and constant power factor for load in bus 3 have been chosen. It was also necessary to recalculate the original network scheme in Fig. 3a) into an equivalent π - element – see Fig. 3b).

The final 2-bus power system (Fig. 3b)) can be described using the load flow nodal equations – see below.

$$\begin{aligned}\frac{p_{ref}-jq_{ref}}{1\cdot e^{j0^\circ}}&=\left(g_{13}+g_{10}+j\left(b_{13}+b_{10}\right)\right)\cdot 1\cdot e^{j0^\circ}-\left(g_{13}+jb_{13}\right)v_B e^{j\theta}\\ \frac{-p_k+jq_k}{v_B e^{-j\theta}}&=-\left(g_{13}+jb_{13}\right)\cdot 1\cdot e^{j0^\circ}+\left(g_{13}+g_{30}+j\left(b_{13}+b_{30}\right)\right)v_B e^{j\theta}\end{aligned}\qquad(1)$$

For bus 3, active and reactive loads with constant power factor (p.f.) can be expressed as follows:

$$q_k=p_k\tan\left(\arccos \text{p.f.}\right)=e\cdot p_k\qquad(2)$$

When separating real and imaginary components, the following system of equations can be obtained from the second equation of (2).

$$\begin{aligned}p_k&=g_{13}v_B\cos\theta+b_{13}v_B\sin\theta-\left(g_{13}+g_{30}\right)v_B^2\\ q_k&=g_{13}v_B\sin\theta-b_{13}v_B\cos\theta+\left(b_{13}+b_{30}\right)v_B^2=ep_k\end{aligned}\qquad(3)$$

Voltage conditions in the network can be examined by solving the above system of equations. Beside the voltage problem, each voltage decrease in bus 3 of the network will result in an increase of voltage phase angle θ, which moves state variables close to the static stability limit.

Relation between the bus voltage v_B and the phase angle θ (Eqn. 4) can be simply derived from Eqn. 3.

$$v_B=\frac{\left(b_{13}+eg_{13}\right)\cos\theta+\left(eb_{13}-g_{13}\right)\sin\theta}{\left(b_{13}+b_{30}\right)+e\left(g_{13}+g_{30}\right)}\qquad(4)$$

When putting Eqn. 18 into the first of Eqn. 3, final formula for static stability $p_k = f(\theta)$ can be obtained.

$$\begin{aligned}p_k=&\frac{\left(g_{13}b_{30}-g_{30}b_{13}\right)\left(b_{13}+eg_{13}\right)}{\left[\left(b_{13}+b_{30}\right)+e\left(g_{13}+g_{30}\right)\right]^2}\cos^2\theta+\frac{\left(b_{13}^2+b_{13}b_{30}+g_{13}^2+g_{13}g_{30}\right)\left(eb_{13}-g_{13}\right)}{\left[\left(b_{13}+b_{30}\right)+e\left(g_{13}+g_{30}\right)\right]^2}\sin^2\theta+\\ &-\frac{2\left(g_{13}+g_{30}\right)\left(g_{13}e+b_{13}\right)\left(b_{13}e-g_{13}\right)-\left(b_{13}^2+2g_{13}b_{13}e-g_{13}^2\right)\left(b_{13}+b_{30}+e\left(g_{13}+g_{30}\right)\right)}{\left[\left(b_{13}+b_{30}\right)+e\left(g_{13}+g_{30}\right)\right]^2}\cos\theta\cdot\sin\theta\end{aligned}\qquad(5)$$

The V-P curve can be found by expressing terms 'cos θ' and 'sin θ' separately from Eqn. 4 using the trigonometrical identity formula. By placing them into the first of Eqn. 3, the nose curve is as follows.

$$p_k=\frac{\begin{gathered}\left(g_{13}^2+b_{13}^2\right)\left[\left(b_{13}+b_{30}\right)+e\left(g_{13}+g_{30}\right)\right]ev_B^2+\\ +\left(g_{13}^2+b_{13}^2\right)v_B\sqrt{\left(g_{13}-eb_{13}\right)^2+\left(b_{13}+eg_{13}\right)^2-\left[\left(b_{13}+b_{30}\right)+e\left(g_{13}+g_{30}\right)\right]^2v_B^2}\end{gathered}}{\left(g_{13}-eb_{13}\right)^2+\left(b_{13}+eg_{13}\right)^2}-\left(g_{13}+g_{30}\right)v_B^2\qquad(6)$$

Values of critical voltage v_{crit} and static stability $-\theta_{crit}$ can be found by differentiating Eqns. 5 and 6 by v_B and θ and setting to zero. The value of maximum loadability p_{kmax} can be then simply obtained from Eqn. 6.

$$\left.\begin{aligned}\frac{dp_k}{dv_B}&=0\rightarrow v_{Bcrit}=0.669511426\,\text{pu}\,(\approx 267.81\,\text{kV})\\ \frac{dp_k}{d\theta}&=0\rightarrow -\theta_{crit}=34^\circ 12'29.32''\,(\approx 0.59705\,\text{rad})\end{aligned}\right\}\rightarrow p_{kmax}\approx 4.9487\,\text{pu}\ (494.87\,\text{MW})\qquad(7)$$

As can be seen from the results, the maximum loadability was reached for the phase displacement value ($-\theta_{crit}$) lower than 45 degrees which is the maximum theoretical static stability limit value for all simple 2-bus power systems. In comparison, the static stability limit of 90 degrees applies only for the generator and its close adjacent area.

Above, the analytical procedure has been introduced for analyzing the voltage stability of simple power systems. However, this approach can be used only for 2-bus and simple 3-bus networks. For more complex and robust systems, the nodal power flow equations become nonlinear and cannot be solved using analytical methods. In other Author's project, the attempts have been placed on the possibility of destrying the nonlinearity by treating voltage phase angles (i.e. the cause of nonlinearity) as constants (possible from phasor metering). Such approach has been tested on a similar network to the one in Fig. 3, where the transmission line was supplied from two independent power sources modelling two separated parts of the entire transmission power system of the country. Final relations for describing all steady-state processes in this network have been successfully derived. However, these formulas for V-P curve, but also for active and reactive power flows, etc. are not presented in this paper due to limited content of this paper and their complexity.

The alternative to the analytical approach is the load flow analysis using numerical methods such as the Gauss-Seidel and the Newton-Raphson. Both of these methods can relatively devotedly simulate the stable portion of the V-P curve. However, both of these methods gain numerical stability problems when approaching the singular point. Especially in the Newton-Raphson method, the Jacobian is becoming singular (det J = 0) when close to the singular point. Due to this reason, the divergence of the Newton-Raphson is inevitable.

For the evaluation of formulas analytically derived above for the 3-bus power system, the Gauss-Seidel has been used since it possesses broad range of numerical stability around the singular point. Below (Fig. 4), the V-P curve along with P-θ and V-θ dependencies is transparently shown. Analytical solutions are depicted by cyan colour, while numerical results are drawn by red, blue and green colours, respectively. Note: Power factor 0.95 has been chosen for the result verification.

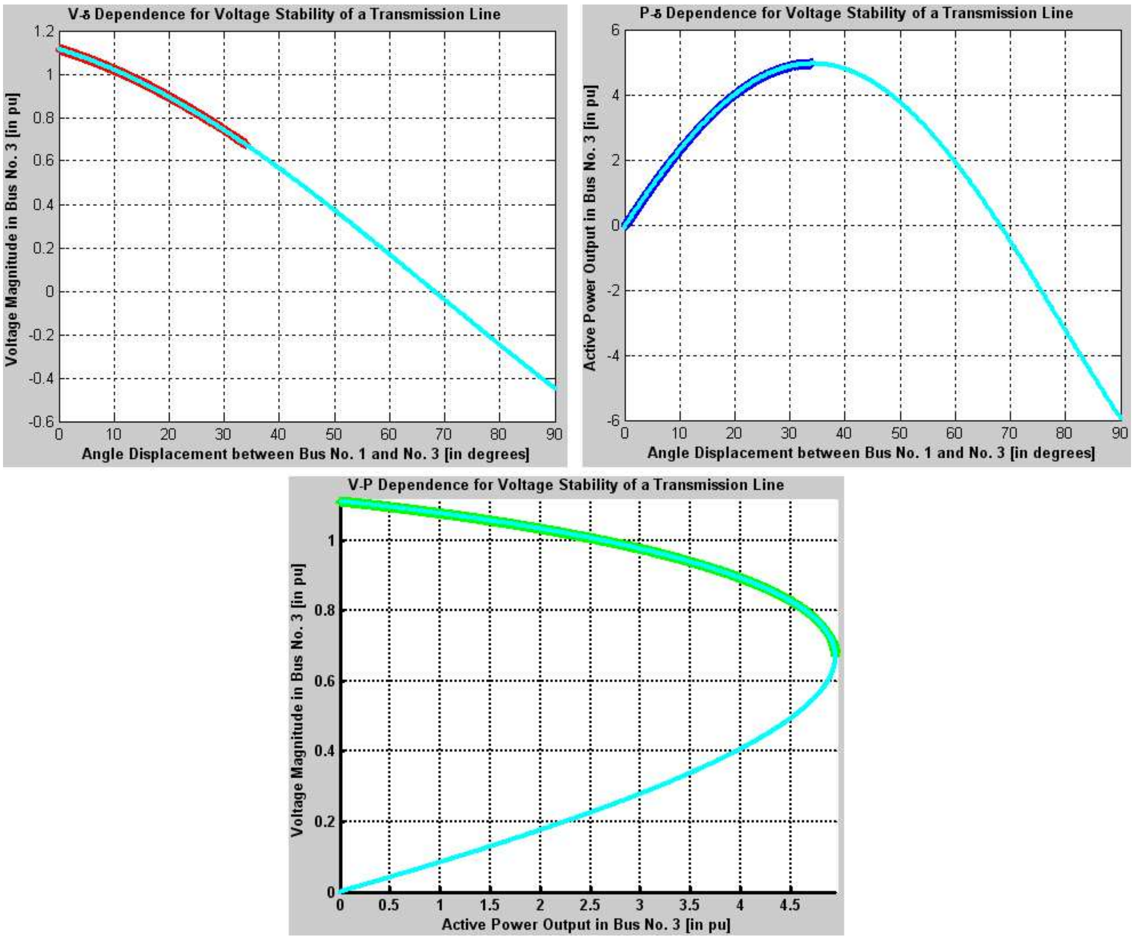

Figure 4 – Final graphical outputs for the 2-bus power system (both analytical and numerical)

As can be seen, results from both analytical and numerical methodologies correspond to each other. For transparency reasons, demanded limit values obtained in numerical way are as follows: p_{max} = 4.948 pu, v_{crit} = 0.676065 pu, $-\theta_{crit}$ = 33° 49' 50.16".

As already known, the Gauss-Seidel method cannot be practically used for solving the load flow problems in large electric power systems due to only linear speed of convergence and large number of iterations which are even strongly dependent on the size of the analyzed network. Due to these factors, the Newton-Raphson method was massively adopted in the past for becoming more suitable for assessing stable and unstable operations of electric power systems.

3. CONTINUATION LOAD FLOW ANALYSIS

Several static approaches consequential from the traditional load flow programs are being used for the evaluation of the network stability – from the optimal active and reactive power flow (OPF) and eigenvalue analysis through the sensitivity based and path-following methods (such as the continuation load flow) up to modal analysis for transient voltage stability.

Continuation load flow analysis suitably modifies conventional load flow equations to become stable also in the singular point of the V-P curve and therefore to be capable to calculate both upper and lower part of the V-P curve. For this, it uses the two-step predictor/corrector algorithms along with single new unknown state variable (so-called continuation parameter). The predictor estimates approximate state variable values in the new step (close to the V-P curve) while the corrector makes the corrections of new state variable values to suit the load flow equations – see Fig. 5. As a result, the voltage stability margin for the current operational point can be simply evaluated. Moreover, based on the differential changes of state variables in each predictor step it is possible to locate weak buses and even areas of the system with largest voltage changes with respect to the load increase.

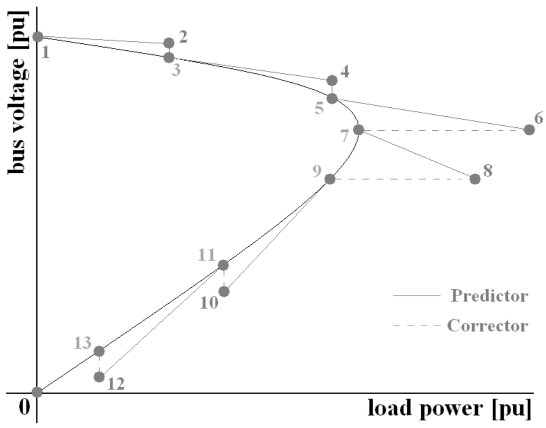

Figure 5 – Continuation load flow analysis (both predictor/corrector steps)

Furthermore, the continuation load flow algorithm can be simply formulated for load increase (both active and reactive) in a single bus, in particular network area with more buses or even in the entire network.

4. INITIAL CONTINUATION LOAD FLOW STUDIES

For the first analysis set, the continuation load flow predictor/corrector algorithm has been created in Matlab environment for simulating voltage stability in a chosen PQ bus of the network and relevant responses to the increase of both active and reactive power loads with constant power factor. The first case study is the power system analytically examined in Chapter 2.2, for which the load power factor in bus 3 was chosen as 0.95. In Fig. 6 below, the comparison between the analytical solution (greed line) and the continuation load flow analysis (blue dots) has been made. As can be seen, both figures loyally imitate each other. Using the numerical approach, maximum loadability of 4.94817 pu and critical voltage of 0.67522 pu have been obtained. When compared to critical values from the analytical solution (4.9784 pu and 0.6695 pu, Eqn. 7), it is obvious that high accuracy of the results has been maintained.

In the second study, the IEEE 14-bus power system has been analyzed in terms of the voltage stability. This network (see Fig. 7) contains in total 8 physical PQ buses (No. 4 and 5 in the 132 kV section, No. 9 to 14 in the 33 kV section).

The continuation load flow analysis for voltage stability in each PQ bus of the network has been performed separately by increasing both active and reactive power loads with maintained power factor (from the original load connected). Final limit values for each PQ bus are shown in Tab. 1. below.

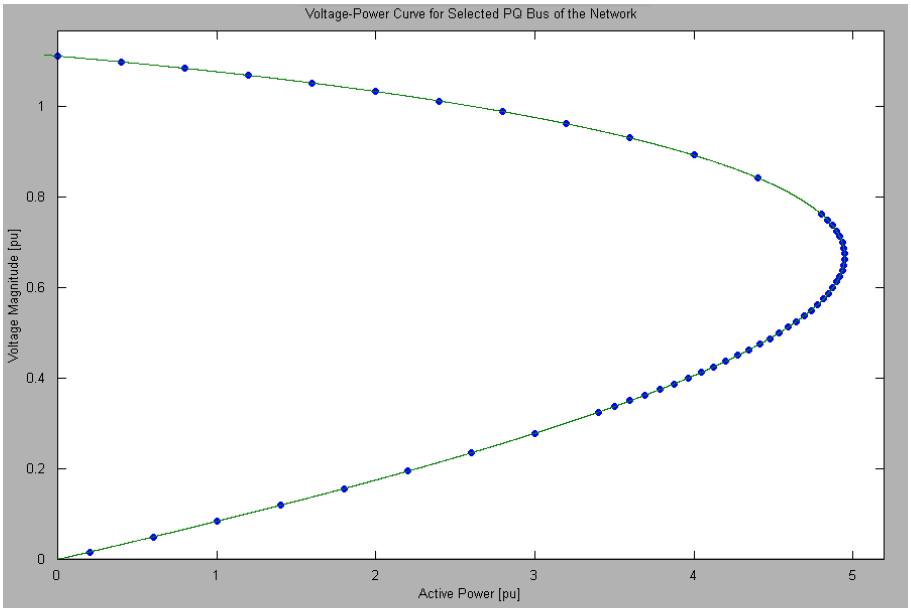

Figure 6 – Comparison of V-P curves from both analytical and numerical approaches (3-bus system from Chapter 2.2)

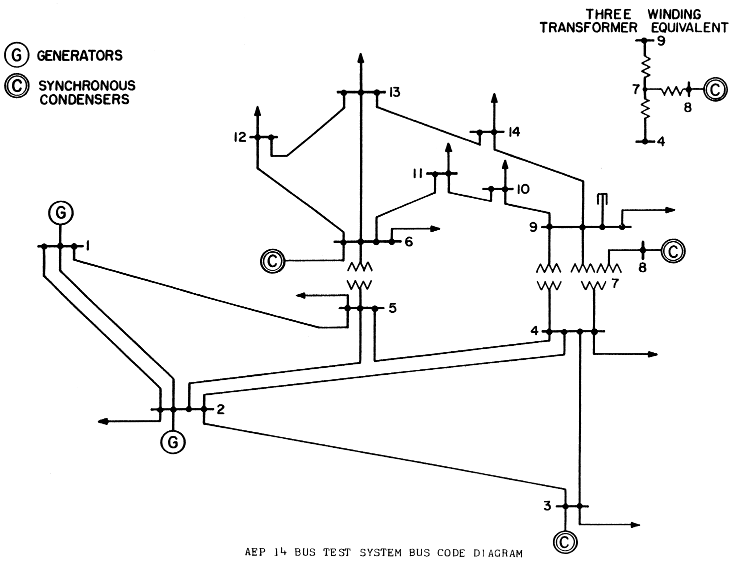

Figure 7 – The IEEE 14-bus power system (downloaded from [8])

As obvious, stability margin values are relatively low which is caused by two main reasons. First, more proper stability analysis would be necessary for increasing all loads in the network simultaneously rather than increasing only the active and reactive power load in one particular bus while keeping constant loads in remaining buses of the system. Second, no var limits for PV buses have been considered at the beginning for making the algorithm simpler. However, during the simulations this fact caused serious convergence problems especially in lower parts of V-P curves for PQ buses connected to some of generation or compensation busbars in the network. Nevertheless, the singular point has been always measured reliably. In other words, Author neglected the influence of possible depletion of reactive power for compensation purposes in the network operation.

Therefore, when considering both of causes above, real maximum loadabilities will be much lower than shown in Tab. 1.

Table 1 – Limit values (loadability and critical voltage) for each PQ bus of the IEEE 14-bus network

Bus No.	P_{Load} [pu]	P_{Limit} [pu]	V_{Crit} [pu]	Margin [%]
4	0.478	7.265646	0.682373	6.57891
5	0.076	6.055156	0.624913	1.25513
9	0.295	2.536249	0.597676	11.63135
10	0.090	1.696699	0.570915	5.30442
11	0.035	1.870922	0.568083	1.87074
12	0.061	1.826136	0.567813	3.34039
13	0.135	2.692317	0.603614	5.01427
14	0.149	1.356010	0.581294	10.98812

5. CONCLUSIONS

Main aim of this paper was to introduce the voltage stability problem along with possible analytical solution. In the second half, the continuation load flow analysis was presented and used for computing critical values (maximum loadability, critical voltage) and the stability margin for a 3-bus and IEEE 14-bus power systems using the developed programming tool in Matlab environment.

In future work, further development of the program will continue for incorporating the full-scale steady-state voltage stability analysis of even larger power systems. The target will be especially to evaluate stability margin values for all PQ buses concurrently with identifying weak areas of the network with respect to the load increase. Second, load flow analysis of ill-conditioned power systems and networks containing bad input data will be performed using the CLF algorithm for locating the errors in input load data and therefore to enable the load flow calculation using the conventional numerical methods such as the Gauss-Seidel and the Newton-Raphson.

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